Stochastic Projection Methods for Social Security Systems

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Abstract

The objectives of *Stochastic Projection Methods for Social Security Systems* are to explain the advantages that may be achieved for Social Security policy purposes by developing stochastic financial projections and to provide a general overview of stochastic projection methodology with particular reference to its application to the Social Security system in the United States.

Keywords

Social Security; Actuarial Uncertainty; Stochastic Projection; Monte Carlo Simulation; Time-series Analysis; R-squared; Vector Autoregression.

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Introduction

Planning and managing the long-term financial requirements for Social Security systems is a major issue for many countries. Financial projections for Social Security systems depend on many demographic, economic and social factors as well as system-specific parameters. Actuarial expertise is required in selecting the values assigned to each of the underlying factors such as mortality, fertility, immigration, and workforce participation, as well as interest rates, inflation, and wage and productivity growth rates. The traditional approach to selecting values for these factors is deterministic; in this approach each factor is assigned a central value that, although it may vary through time, is held constant at each point in time without recognizing its potential variability from the central value. The disadvantage of this deterministic methodology is that the user is provided with only a single financial projection without any indication of the range of plausible outcomes that might vary from the deterministic projection. However, a feature of deterministic projections that may be perceived as an advantage is that the value of each factor is selected and determined in advance to define the characteristics of a unique scenario.

Some Social Security systems utilize multiple scenario approaches to indicate a range of plausible outcomes for financial projections. In the United States, for example, alternative financial projections, based on a set of three different scenarios, are developed to comprise a best estimate together with a high-cost estimate and low-cost estimate. These are essentially deterministic scenarios that provide a range of future plausible outcomes. However, these alternative deterministic scenarios may, in practice, have some limitations for policy purposes, since they do not indicate any associated prospective quantitative measure of the likelihood of being realized; that is to say, that the user does not know, for example, whether the alternative scenarios have a 25% or a 1% or some other probability of being realized in practice.

In order to address the need to develop an entire range of plausible outcomes with associated quantitative measures of the likelihood of being realized, stochastic methods are being developed and applied to produce financial projections for Social Security systems in some countries. These stochastic projection methods add significantly to the information provided, and have certain advantages over deterministic projection methods, since they provide the user with an entire range of plausible future outcomes that diverge from the simple deterministic projection and, moreover, if designed appropriately, provide an associated measure of the likelihood of being realized. The user of stochastic projection results is thus enabled to assess the level and range of future projected results for a Social Security system and see the difference in results with, for example, a 50%, 95% and 5%, and 99% and 1%, likelihood of being realized in practice.

The application of stochastic projection methodology is complex since it involves the identification and selection of mathematical formulae or parameters that define the probability distributions of each of the fundamental underlying demographic, economic and social factors as well as the effect of secular trends and the inter-relationships between the factors. For example, stochastic projections of mortality rates can recognize not only a secular trend in improving mortality experience reflecting lower rates over time, but also a range of plausible outcomes around both the current level and the central secular trend of declining rates. Another area of complexity in stochastic projection models is the need to recognize the extent to which certain factors are inter-related and have a direct impact on each other, for example, interest rates, inflation and wage growth rates typically demonstrate a degree of correlation and do not move independently of each other over time.

Principle of Actuarial Uncertainty

The work of actuaries is typically concerned with the projection of asset and liability cash flows for insurance companies, pension funds and social security systems. But these asset and liability cash flows cannot be projected with absolute certainty; there is an inherent element of uncertainty associated with any actuarial projection. In *Principles Underlying Asset Liability Management* [vide: reference {1}] the Society of Actuaries Task Force on Asset Liability Management Principles identified the dynamic environments in which these institutional entities operate, as well as the effects of pure randomness, as factors that create uncertainties in the measures of cash flows and, hence, in the true future risk exposures of the entity. Risk varies as the underlying risk factors, such as interest rates and mortality, change in a dynamic manner over time, with either positive or negative financial implications, and as the original estimated future expected cash flows are replaced by the results emerging from the actual cash flows that are experienced. This process, reflecting how cash flows react to factor changes will often result, over time, in revisions being made to future risk factor assumptions. The management and oversight of the financial condition of Social Security systems will require the use of models to project the systems' future uncertain cash flows. In many cases, simple deterministic models are used and the management and oversight activities are based on one single projection of expected future cash flows. In other cases, such as when future cash flows are expected to depend on future economic conditions, more complex stochastic models (or multiple sets of deterministic scenarios) may be required to understand the interaction of the various factors that affect future cash flows. Stochastic models are often used to simulate future expected cash flows under various scenarios to help identify the associated risk exposures. These models produce statistical distributions of potential results and, as a consequence, different management strategies and policy options can be evaluated by studying the range of results produced from modeling these cash flow projections. Modeling can also be used to construct many possible futures or scenarios, and then, results across all the scenarios can be used to measure risks inherent in the system. There are also certain risks associated with the use of a particular model; model risk is created when the model does not adequately represent the underlying process or reality. There are two general classes of model risk; the risk of model misspecification or oversimplification, and the risk of a changing environment not anticipated in the model. Using a particular model with a too simple characterization of the distribution of an economic variable may not disclose the inherent risk to the system. Model risk would, for example, exist in circumstances where an inappropriate model is used with too few extreme value sample points to adequately assess the risk for extreme events that have a low incidence but significant financial impact. In addition, the volatility of certain economic factors may vary over time and this may not be accurately captured in the model. Communicating the nature of uncertainty requires particular care and skill to ensure that the intended users of actuarial projections adequately comprehend the implications of reliance on single-scenario or multi-scenario deterministic projections. Even greater care and skill is required in communicating the uncertainties and the degree of credibility associated with stochastic projections.

Advantages of Stochastic Projection Methods

Many users of actuarial projections intuitively interpret them as absolute measures of future outcomes without an explicit acknowledgment of the uncertainties involved or the stochastic nature of the various risk elements underlying the projections. Historically, conventional actuarial practice has summarized the expected future cash flows into a single number as a discounted present value. This process, which conveys the actuarial value of a series of future contingent cash payments, by converting them into a simple, easy-to-understand, single number, has the effect of masking the extent of the uncertainty associated with this discounted present value. Actuarial projections are typically re-evaluated on a dynamic basis at regular intervals so as to capture the extent of deviations of the actual emerging experience from that assumed in the original or preceding projection values. This process often reveals unanticipated financial risks that were inherent in the principle of actuarial uncertainty. In recent years the use of stochastic projection techniques has gained wider acceptance, replacing or supplementing traditional deterministic projection techniques. Stochastic projections produce an array of plausible outcomes with an associated probability or credibility measure. While a single deterministic projection or the 50th percentile of a stochastic projection may represent a best estimate at a single point in time, the stochastic projection model permits the user to see a full array of potential outcomes that deviate from the best estimate value. Some proponents of deterministic projection methodology argue that the use of stochastic models to present an array of results with associated probabilities is confusing and difficult for the typical user or policy-maker to comprehend and consequently advocate the use of simple deterministic projections only. However, advocates of stochastic methodology believe there is great merit in presenting and carefully explaining the results of stochastic projections as a means of illustrating the nature of uncertainty and the quantification of risk associated with the interpretation of actuarial projections of future contingent events.

The United States Social Security System and Measures of Financial Status

The US Social Security system was created when the US Congress passed the Social Security Act in 1935. Initially, the system, which became effective in 1937, provided retirement income benefits to workers age 65 and older. The system was expanded in 1939 to cover dependents and survivors and became the Old-Age and Survivors Insurance (OASI) program. Then in 1956 the Disability Insurance (DI) program was added to provide income to disabled workers, and in 1958 it was extended to provide benefits to dependents of disabled workers. The US Social Security system is financed by payroll taxes assessed equally on employers and employees. For 2007, OASI payroll taxes are set at a combined rate of 10.6% of earnings up to a limit of US$97,500. The corresponding DI payroll tax rate for 2007 is 1.8%, making the combined OASDI tax rate 12.4%. The rates for the self-employed are somewhat less in total.

The financial condition of the US Social Security system, comprising the OASI and the DI trust funds, is presented in the annual reports of the Social Security Board of Trustees. Each year, the trustees present a report [vide: reference {2}] on the financial operations of the trust funds, including assumptions about the future, and resultant projections of the future financial status of the system. The trustees present the results of long-range actuarial estimates, extending up to 75 years, of the annual income rates, cost rates and balances for the OASI trust fund, the DI trust fund, and the combined OASDI funds. For the purpose of preparing the long-range actuarial estimates, the Social Security actuaries utilize demographic assumptions and models relating to such factors as mortality, fertility and immigration, to develop total population estimates. As a part of this process, they also utilize economic assumptions including productivity, inflation, average earnings, real-wage differentials, the labor force, unemployment, gross domestic product and interest rates.

After projecting the funds' income, expenditure and assets at various future points of time over the next 75 years, the Social Security actuaries present the results in terms of annual income rates, cost rates and balances. The annual income rate is the ratio of income from revenues, comprising payroll tax contributions and income from the taxation of benefits, to the OASDI taxable payroll for the year. The annual cost rate is the ratio of the cost, comprising outgo and expenditures for benefits, administrative expenses and other disbursements, of the program, to the taxable payroll for that year. In this context, the balance is simply the difference between the income rate and the cost rate for a specific year.

The next step in preparing the results of the 75-year projections is the development of summarized income rates, cost rates and balances. The summarized rates represent the projected annual rates on a present-value basis for various periods within the overall 75-year projection period. Results are developed for 25-year, 50-year and 75-year projection periods, representing cash flows from income and costs, without having regard to the initial trust fund balance, any minimum target level for the trust fund assets, or the adequacy of the trust fund to meet scheduled benefit payments. Then the summarized income rates and cost rates are adjusted to include the effect of the initial trust fund balance and to maintain a minimum target trust fund balance equal to one year's outgo for benefits and expenses at the end of the projection period. The difference between the summarized income rates and summarized cost rates with these trust fund adjustments is referred to as the actuarial balance. This is a measure of the surplus or deficit in the system and is widely regarded as an important quantitative measure of its financial viability.

Methodology for Stochastic Projections of the United States Social Security System

In 2004 the Office of the Chief Actuary of the US Social Security Administration developed a stochastic model for projecting the principal financial measures of the system. The model is described in detail in *Actuarial Study No. 117: A Stochastic Model of the Long-Range Financial Status of the OASDI Program* [vide: reference {3}]. This stochastic model was built on a foundation of time series and Monte Carlo simulations for a number of demographic and economic assumptions. The specific assumptions for which the model incorporates stochastic features are: fertility, mortality, immigration, unemployment, inflation, real interest, real average wage growth, and disability incidence and recovery. The stochastic model was designed to be consistent with the corresponding deterministic model by requiring the projected values for each variable to equal those for the deterministic best-estimate assumptions (referred to as the intermediate assumptions) in the absence of variation. Stochastic variation is incorporated in the model by means of various equations based on standard time series models. Generally these equations include the variable's prior period values, prior period error terms, other variables and a random error term. In applying autoregressive (AR) or autoregressive moving average (ARMA) methodology to develop the appropriate time series, the nature and quality of the historical data were considered in setting the choice of ranges for the regressions. The following paragraphs describe the approach taken with respect to each of the stochastically related assumptions.

With respect to fertility, time-series analysis was applied to the total fertility rate as represented by the sum of age-specific birth rates for women aged 14 through 49. Historical data for the total fertility rate in the United States from 1917 onwards are available from the National Center for Health Statistics and the US

Census Bureau. An ARMA (4,1) equation was applied with the parameters estimated using the entire range of data.

With respect to mortality, time-series analysis was applied to the annual rate of decrease in the central death rate. Central death rates are calculated for 42 age-sex groups for the period from 1900 onwards. Data for the annual numbers of deaths are available from the National Center for Health Statistics and the data for the size of the resident population are available from the US Census Bureau. For the population aged 65 or older, data beginning in 1968 are from the Centers for Medicare and Medicaid Services. An AR(1) equation was applied for the annual rate of decrease in the central death rate for each age-sex group.

With respect to immigration, time-series analysis was applied separately to legal immigration, legal emigration and other net immigration. Data for legal immigration are available from 1901 onwards from the US Citizenship and Immigration Services. An ARMA (4,1) equation was applied with the parameters estimated using the entire range of historical data for legal immigration. Although emigration data are not collected in the United States, the US Census Bureau estimates that emigration has historically totaled approximately 25% of legal immigration. For the purposes of the stochastic model, the same time-series equation was applied for emigration as for legal immigration with the parameters reduced by 75%. Because data does not exist for net other immigration, reliance is placed on indirect measurements from consecutive decennial census populations recorded by the US Census Bureau; this is accomplished by isolating known components of population change and assigning the residual numbers to other net immigration. The annual level of other net immigration is assumed to follow a random walk.

With respect to unemployment, inflation and real interest, these rates are simulated together using a vector autoregression so as to capture the economic relationships between the three variables. In the vector autoregression, each variable is regressed on the prior-period values of all three variables. Based on tests of reasonableness of fit for different prior-period lengths, a vector autoregression including two prior years was used. Historical data for the period 1960 onwards was reviewed and evaluated as a part of the process. For the vector autoregression, the unemployment rates were expressed as log-odds ratios to bound the values between 0 and 100 percent. A logarithmic transformation was applied to the adjusted inflation rates to provide a lower bound for the vector autoregression. An amount of 3.0 percent was added to the inflation rate series prior to the log-transformation to provide a lower bound of minus 3.0 percent. The respective R-squared values for the unemployment rate, inflation rate and real interest rate were 0.85, 0.83 and 0.81 respectively.

With respect to real average wage growth, the model is concerned with real average covered wage that is defined as the ratio of the average nominal OASDI covered wage to the adjusted inflation rate. Although the annual growth rate in the real average covered wage has differed significantly from the annual growth rate in a real average economy-wide wage series due to historical changes in covered employment, these two measures are expected to be almost identical in the future. The historical variation of the economy-wide wage is used to model the future variation in the real average covered wage. The real average economy-wide wage is the ratio of the average nominal wage to the adjusted Consumer Price Index. The nominal wage is the ratio of wage disbursement to the total Civilian Employment together with numbers for the US Armed Forces. The model estimates the annual percent changes in the real economy-wide wage as a function of the current and prior year unemployment rate, expressed as log-odds ratios, over the period from 1968 onwards.

With respect to disability, the stochastic model generates separate equations for the disability incidence rate and the disability recovery rate. The disability incidence rate is the proportion of the exposed population at the beginning of a year who become newly entitled to disability benefits during the year. Data on disability incidence are from the records of the Social Security Administration and these are age-adjusted to a 1996 reference exposed population. The equations for disability incidence rates are selected separately for males and females. Using time-series analysis, both series were modeled as AR(2) processes. The disability recovery rate is the proportion of disabled-worker beneficiaries whose disability benefits terminate as a result of the individual's recovery from disability. The disability recovery rates were modeled in a similar fashion to the disability incidence rates, but using an AR(1) process.

The stochastic projection model developed by the Office of the Chief Actuary of the Social Security Administration was written in FORTRAN 90/95 and compiled using Intel Visual FORTRAN Compiler. The program has about 26,000 lines of source code and was written in modular format with 20 source code files; it uses more than 160 data files as input; it consists of nine modules that are executed sequentially in the following order: Assumptions; Population; Economics; Insured; Disability Insurance Beneficiaries; Old-Age and Survivors Insurance Beneficiaries; Awards; Cost; and Summary Results. Certain risks such as statistical process risk, cyclical risk, parameter uncertainty risk, and incorrect data risk are not explicitly taken into account by the model. These risks, if incorporated into the model would increase the resulting range of uncertainty. Discontinuities or structural shifts, as used in regime switching techniques, are not taken into account by the model. If certain changes are made to the model specifications then the projection results might be significantly affected; the results from the model would be sensitive to changes in the specification of any of the equations or to modifications in the degree of interdependency among economic variables. In addition, if new variables were modeled stochastically, such as labor force participation rates, retirement rates, marriage rates and divorce rates, the current stochastic projected variation would be increased.

An Alternative Stochastic Projection Model

The US Congressional Budget Office (CBO) has developed an alternative stochastic projection model for the US Social Security system that differs in important respects from that of the Office of the Chief Actuary of the Social Security Administration. The CBO stochastic projection model uses a variety of techniques for different parts of the projection and is described in detail in the CBO Background Paper: *Quantifying Uncertainty in the Analysis of Long-term Social Security Projections* [vide: reference {4}]. The CBO model includes the use of microsimulation methods. Its modules are embedded within a government-wide budget and growth model framework. CBO projects population size and its composition by age and sex using the same techniques involving fertility, mortality and immigration as for the stochastic model of the Office of the Chief Actuary of the Social Security Administration. The CBO projections of overall economic activity and the finances of the Social Security system are based on an integrated macroeconomic and microeconomic framework. A difference arising from this approach is in economy-wide real wage growth assumption, an important determinant of both aggregate system finances and individual taxes and benefits. In the CBO projection model, real wage growth is determined by the combination of several assumptions, most notably total factor productivity, labor force participation and capital accumulation. Total factor growth is modeled as a white noise process. The CBO model utilizes a vector autoregression model for the unemployment rate, inflation rate, and the real interest rate gap that represents the gap between the average product of capital and the ten-year interest rate.

Conclusion and Recommendation

Although it is recognized that circumstances may differ widely from country to country with respect to the nature of Social Security systems, the availability of relevant demographic and economic data, and the requirements of policymakers for financial projections, it is evident that there are advantages to be gained from applying stochastic methodology to generate financial projections as compared to deterministic methodology. The use of stochastic model methodology can provide valuable analysis and insights into the range of plausible outcomes for long-term projections of the finances of Social Security systems and provides associated measures of the probability of a specific outcome being realized in practice. Accordingly, it is recommended that in those countries where the financial projections of the Social Security system are based on deterministic methodology, consideration be given to the potential advantages from developing the capacity to generate stochastic models to apply to the generation of long-range financial projections.

References

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